

**UNIT-IV: UNCERTAINTY****Uncertainty**

[ To act rationally under uncertainty we must be able to evaluate how likely certain things are.

[ With FOL a fact F is only useful if it is known to be true or false.

[ But we need to be able to evaluate how likely it is that F is true.

[ By weighing likelihoods of events (probabilities) we can develop mechanisms for acting rationally under uncertainty.

**Dental Diagnosis example.**

[ In FOL we might formulate

[ P. symptom(P,toothache) →  
disease(p,cavity)    disease(p,gumDisease)  
disease(p,foodStuck)    L

[ When do we stop?

[ Cannot list all possible causes.

[ We also want to rank the possibilities. We don't want to start drilling for a cavity before checking for more likely causes first.

**Axioms Of Probability**

[ Given a set U (universe), a probability function is a function defined over the subsets of U that maps each subset to the real numbers and that satisfies the Axioms of Probability

1.  $\Pr(U) = 1$

2.  $\Pr(A) \in [0,1]$

3.  $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

Note if  $A \cap B = \{\}$  then  $\Pr(A \cup B) = \Pr(A) + \Pr(B)$

### **BASIC PROBABILITY NOTATION**

- 1 Unconditional or prior probabilities
- 2 Conditional or posterior probabilities

### **SEMANTICS OF BAYESIAN NETWORK**

- 1 Representation of joint probability distribution
- 2 Conditional independence relation in Bayesian network

### **INFERENCE IN BAYESIAN NETWORK**

- 1 Tell
- 2 Ask
- 3 Kinds of inferences
- 4 Use of Bayesian network

### **TEMPORAL MODEL**

- 1 Monitoring or filtering
- 2 Prediction

### Bayes' Theorem

Many of the methods used for dealing with uncertainty in expert systems are based on Bayes' Theorem.

#### Notation:

$P(A)$  Probability of event A

$P(A \cap B)$  Probability of events A and B occurring together

$P(A | B)$  Conditional probability of event A  
given that event B has occurred

If A and B are independent, then  $P(A \cap B) = P(A) \cdot P(B)$ .

Expert systems usually deal with events that are not independent, e.g. a disease and its symptoms are not independent.

#### Theorem

$P(A \cap B) = P(A | B) \cdot P(B) = P(B | A) \cdot P(A)$  therefore  $P(A | B) = P(B | A) \cdot P(A) / P(B)$

Uses of Bayes' Theorem

In doing an expert task, such as medical diagnosis, the goal is to determine identifications (diseases) given observations (symptoms). Bayes' Theorem provides such a relationship.

$P(A | B) = P(B | A) \cdot P(A) / P(B)$

Suppose: A = Patient has measles, B = has a rash

Then:  $P(\text{measles} | \text{rash}) = P(\text{rash} | \text{measles}) \cdot P(\text{measles}) / P(\text{rash})$

The desired diagnostic relationship on the left can be calculated based on the known statistical quantities on the right.

#### Joint Probability Distribution

Given a set of random variables  $X_1 \dots X_n$ , an atomic event is an assignment of a particular

value to each  $X_i$ .

The joint probability distribution is a table that assigns a probability to each atomic event. Any question of conditional probability can be answered from the joint. [Example from Russell & Norvig.]

	Toothache	$\neg$ Toothache
Cavity	0.04	0.06
$\neg$ Cavity	0.01	0.89

### Problems:

- The size of the table is combinatoric: the product of the number of possibilities for each random variable.
- The time to answer a question from the table will also be combinatoric.
- Lack of evidence: we may not have statistics for some table entries, even though those entries are not impossible.

### Chain Rule

We can compute probabilities using a chain rule as follows:

$$P(A \ \&\& \ B \ \&\& \ C) = P(A \mid B \ \&\& \ C) * P(B \mid C) * P(C)$$

If some conditions  $C_1 \ \&\& \ \dots \ \&\& \ C_n$  are independent of other conditions  $U$ , we will have:

$$P(A \mid C_1 \ \&\& \ \dots \ \&\& \ C_n \ \&\& \ U) = P(A \mid C_1 \ \&\& \ \dots \ \&\& \ C_n)$$

This allows a conditional probability to be computed more easily from smaller tables using the chain rule.

## Bayesian Networks

Bayesian networks, also called belief networks or Bayesian belief networks, express relationships among variables by directed acyclic graphs with probability tables stored at the nodes.[Example from Russell & Norvig.]

- 1 A burglary can set the alarm off
- 2 An earthquake can set the alarm off
- 3 The alarm can cause Mary to call
- 4 The alarm can cause John to call

## Computing with Bayesian Networks

If a Bayesian network is well structured as a poly-tree (at most one path between any two nodes), then probabilities can be computed relatively efficiently.

One kind of algorithm, due to Judea Pearl, uses a message-passing style in which nodes of the network compute probabilities and send them to nodes they are connected to.

Several software packages exist for computing with belief networks.

A Hidden Markov Model (HMM) tagger chooses the tag for each word that maximizes:  
[Jurafsky, op. cit.]  $P(\text{word} \mid \text{tag}) * P(\text{tag} \mid \text{previous } n \text{ tags})$

For a bigram tagger, this is approximated as:  
 $t_i = \text{argmax}_j P(w_i \mid t_j) P(t_j \mid t_{i-1})$

In practice, trigram taggers are most often used, and a search is made for the best set of tags for the whole sentence; accuracy is about 96%.



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**GKMCET  
Lecture Plan**

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